

Semiclassical description of multiple giant dipole resonance excitation and decay*

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A semiclassical description of multiple giant resonance excitation and decay that incorporates incoherent fluctuation contributions of the Brink-Axel type is developed. Numerical calculations show that the incoherent contributions are important at low to intermediate bombarding energies.

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We wish to develop a schematic semiclassical description of the collective-statistical theory of multiple giant dipole resonance excitation and decay given in Refs. [1] and [2]. To do this, we use the formalism developed in Ref. [3] to extend the semiclassical description of collective multiple giant resonance excitation given in Ref. [4]. This development is important for the understanding of the recent observation and analysis of the heavy-ion Coulomb excitation of the Double Giant Dipole Resonance reported in Refs. [5] and [6].

As in Ref. [1], we wish to take into account the contribution to the cross section of collective nuclear excitations that occur both before and after the statistical decay of earlier collective excitations. We thus label the states of the nucleus with both a collective index n , denoting the number of collective dipole phonons, and a statistical one s , denoting the number of collective phonons that have decayed into the incoherent background. The class of states denoted by the pair of indices n and s thus possesses n phonons of collective excitation and an incoherent background excitation obtained through the decay of another s phonons. We will represent this class of states by a single state. In the limit of harmonic phonons, this state would have an excitation energy of $(n + s)\varepsilon_d$ and a width of $n\Gamma_d$, where ε_d is the energy of the giant dipole resonance and Γ_d is its spreading width. We will neglect contributions of the escape widths, as these are extremely small when compared to those of the spreading widths of the systems of interest here. [7]

Although the collective excitation of the nucleus is a coherent process, its statistical decay is an incoherent one. The time evolution of the system thus possesses both coherent and incoherent aspects, making a density matrix formulation necessary. We consider the time evolution of the density matrix elements $\rho_{ns,n's'}(t)$. The diagonal element $\rho_{ns,ns}(t)$ represents the instantaneous occupation probability of the state with n collective phonons and a statistical background equivalent to s phonons.

Following Ref. [3], we can put the time-evolution equation of the semiclassical density matrix into the form

$$\begin{aligned} \hbar \frac{\partial \rho_{ns,n's'}}{\partial t} = & -i \sum_m \{(\varepsilon_n + \varepsilon_s) \delta_{nm} + V_{nm}(t)\} \rho_{ms,n's'} \\ & + i \sum_m \rho_{ns,ms'} \{(\varepsilon_{n'} + \varepsilon_{s'}) \delta_{mn'} + V_{mn'}(t)\} \\ & - \frac{(\Gamma_{ns} + \Gamma_{n's'})}{2} \rho_{nsn's'} + \delta_{nn'} \delta_{ss'} \sum_{m,r} \Gamma_{ns \leftarrow mr} \rho_{mr,mr}, \end{aligned} \quad (1)$$

The terms in the first two lines on the right-hand side induce the coherent contribution to the evolution. This is given in terms of the (diagonal) collective and statistical contributions to the excitation energy, ε_n and ε_s , and of the interaction V , which couples the states through collective excitation alone. The two terms on the third line describe, respectively, the loss of probability due to incoherent, statistical transitions out of the state and the gain of probability due to statistical transitions from the other states. The partial gain widths $\Gamma_{ns \leftarrow mr}$ are such that

$$\sum_{n,s} \Gamma_{ns \leftarrow mr} = \Gamma_{mr}. \quad (2)$$

This condition simply states that the sum of the partial widths for probability transfer from any one state to all others must be equal to the total width for probability loss from the initial state. This guarantees probability conservation during the evolution of the system (assuming, of course, that V is Hermitian).

We will assume that the initial population is in the ground state. The initial conditions for which the equation will be solved are then

$$\rho_{ns,n's'}(t \rightarrow -\infty) \rightarrow \delta_{nn'}\delta_{n0}\delta_{ss'}\delta_{s0}(1 - T(b)), \quad (3)$$

where $T(b)$ is an impact-parameter dependent transmission coefficient that takes into account the probability of projectile-target interactions more complex than those being discussed here. We approximate the transmission coefficient as

$$T(b) = \frac{1}{1 + \exp((R - b)/a)}, \quad (4)$$

where we take the strong-interaction radius to be $R = 1.23(A_P^{1/3} + A_T^{1/3})$ fm and the diffusivity to be $a = 0.75$ fm, with A_P and A_T the projectile and target mass numbers, respectively.

As the only coherent coupling in the time-evolution equation is through the collective interaction V , which couples only collective states having the same statistical index s , we conclude that the density matrix will remain diagonal in the statistical index s at all times,

$$\rho_{ns,n's'}(t) = \delta_{ss'}\rho_{ns,n's}(t). \quad (5)$$

The density matrix thus reduces to a separate density submatrix for each value of the statistical index, with the coupling between these submatrices, through the gain and loss terms, being completely incoherent.

It is convenient to explicitly take into account the time dependence due to the collective excitation energy. To do this, we define a modified density matrix, which will have the same diagonal matrix elements as the original one, as

$$\rho_{nn'}^s(t) = \exp[-i(\varepsilon_n - \varepsilon_{n'})t/\hbar] \rho_{ns,n's}(t). \quad (6)$$

The time evolution equation then reduces to the form

$$\begin{aligned} \hbar \frac{\partial \rho_{nn'}^s}{\partial t} = & -i \sum_m \left(\tilde{V}_{nm}(t) \rho_{mn'}^s - \rho_{nm}^s \tilde{V}_{mn'}(t) \right) \\ & - \frac{(\Gamma_{ns} + \Gamma_{n's})}{2} \rho_{nn'}^s + \delta_{nn'} \sum_{r,m} \Gamma_{ns \leftarrow mr} \rho_{mm}^r, \end{aligned} \quad (7)$$

in which the remaining contribution to the coherent evolution is due to \tilde{V} , where

$$\tilde{V}_{nn'}(t) = \exp[i(\varepsilon_n - \varepsilon_{n'})t/\hbar] V_{nn'}(t). \quad (8)$$

The second line of Eq.(7) contains the incoherent contributions of the statistical loss and gain terms, respectively.

Assuming that the collective excited states are harmonic n -phonon giant dipole states, the interaction matrix elements can be written as

$$\tilde{V}_{nn'}(t) = (\exp[i\varepsilon_d t/\hbar] \sqrt{n} \delta_{n',n-1} + \exp[-i\varepsilon_d t/\hbar] \sqrt{n+1} \delta_{n',n+1}) V_{01}(t) \quad (9)$$

where ε_d is the excitation energy of the giant dipole resonance and $V_{01}(t)$ is the semiclassical matrix element coupling the ground state to the giant resonance, which we take to have the simple form

$$V_{01}(t) = V_0 \frac{(b_{\min}/b)^2}{1 + (\gamma vt/b)^2}, \quad (10)$$

as given in Ref. [4]. As is done there, we neglect the spin degeneracies and magnetic multiplicities of the giant resonance states and approximate the projectile-target relative motion as a straight line.

The decay widths in the case of harmonic phonons can be approximated as

$$\Gamma_{ns} = n\Gamma_d, \quad (11)$$

where Γ_d is the spreading width of the giant dipole resonance. We have neglected the contribution to the width of the hot statistical background of states since, at the low temperatures involved here, the decay widths of the hot Brink-Axel resonances are very similar to those of the cold ones.

According to the convention we have adopted for labeling states, the statistical index denotes the number of collective phonons that have decayed to the incoherent statistical background. The decay of the n -phonon s -background state thus transfers its occupation probability to the $(n-1)$ -phonon $(s+1)$ -background state. The form of the gain terms reflects this fact,

$$\Gamma_{ns \leftarrow mr} = \delta_{s,r+1} \delta_{n,m-1} \Gamma_{mr} = \delta_{s,r+1} \delta_{n,m-1} m \Gamma_d. \quad (12)$$

We observe, from the form of the time-evolution equation, that all states will eventually decay to the states containing no collective excitations. We thus have for the asymptotic occupation probabilities,

$$\rho_{00}^s(t \rightarrow \infty) \rightarrow P(n=0, s), \quad (13)$$

with the $P(n=0, s)$ being defined by this limit. All other matrix elements tend to zero,

$$\rho_{nm}^s(t \rightarrow \infty) \rightarrow 0 \quad n, m \neq 0. \quad (14)$$

Conservation of probability requires that

$$\sum_s P(n=0, s) = 1 - T(b), \quad (15)$$

where $T(b)$ is the transmission coefficient of Eq.(4).

Although the states containing collective phonons are asymptotically depopulated, we can still obtain an estimate of the probability that passes through them by calculating the probability that decays out of them. We thus define for these states

$$P(n \neq 0, s) \equiv \Gamma_{ns} \int_{-\infty}^{\infty} dt \rho_{nn}^s(t). \quad (16)$$

We note that this is only an estimate of the probability that has passed through each state, as it takes into account only that part of the probability that decays incoherently. It does not include the fraction of the probability that was transferred coherently (through the action of V) to other states.

Finally, we define a cross section σ_{ns} for each state by integrating its probability $P(n, s)$ over the implicit dependence on the impact parameter,

$$\sigma_{ns} \equiv 2\pi \int_{b_{min}}^{\infty} b db P(n, s). \quad (17)$$

At extremely low energies, the lower limit of the integral over impact parameter, b_{min} , is determined by the classical point of closest approach of the Coulomb interaction. When the energy is sufficiently high to surpass the Coulomb barrier, the transmission coefficient $T(b)$ cuts the integral off at low values of the impact parameter.

We have performed calculations of multiple giant dipole resonance excitation within the model for the system $^{208}\text{Pb} + ^{208}\text{Pb}$ in the projectile energy range from 100 to 1000 MeV/nucleon. For the centroid and width of the giant dipole resonance, we use values taken from a global systematic, $\varepsilon_d = 43.4 A^{-0.215}$ MeV and $\Gamma_d = 0.3 \varepsilon_d$, [8] giving $\varepsilon_d = 13.8$ MeV and $\Gamma_d = 4.1$ MeV, slightly above the experimental values.

We display in Fig. 1, as a function of the projectile energy, the coherent n -phonon cross sections σ_{n0} (dashed lines) and total n -phonon cross sections σ_{0n} (solid lines) obtained from the calculation. The coherent cross sections σ_{n0} describe the direct excitation of the n -phonon states. These are the cross sections that result from a typical calculation of multiple giant resonance excitation amplitudes. The total n -phonon cross sections σ_{0n} account for all possible n -phonon excitations, including those in which one or more of the phonons decays incoherently before others are excited. The respective coherent or total cross sections decrease by about an order of magnitude for each additional phonon of excitation. The coherent n -phonon cross sections increase monotonically with energy, as do the total excitation cross sections for low phonon number. For the cases of three or more phonons, the total n -phonon cross section first decreases with the incident energy, but then turns and increases like the other cross sections.

Except for the 1-phonon case, the total n -phonon cross sections σ_{0n} in Fig. 1 are clearly larger than the coherent cross sections σ_{n0} . This can be readily understood by noting that, although there is only one way a single phonon can be excited and decay, alternative sequences of excitation and decay are available when more than one phonon is involved. As we have emphasized previously in the case of 2-phonons [1,2,7], the total cross section σ_{02} contains both the coherent 2-phonon excitation, 2-phonon decay contribution σ_{20} and an incoherent contribution due to the excitation (and decay) of a second phonon after the first phonon has decayed into the statistical background. The

apparent discrepancy in experimental double giant dipole resonance cross sections [5,6] can thus be explained by arguing that what is observed is the total 2-phonon cross section σ_{02} and not just the coherent cross section σ_{20} .

The relative importance of the coherent excitation cross sections, σ_{n0} , compared to the total ones, σ_{0n} , can best be seen by looking at their ratio, σ_{0n}/σ_{n0} , as shown in Fig. 2 as a function of the projectile energy. We observe that the total n -phonon cross sections σ_{0n} are greatly enhanced relative to the coherent cross sections σ_{n0} at low energies. As the energy increases, the relative enhancement decreases and tends toward one. This trend can be explained by comparing the time scale of the collision process to that of the decay of a giant resonance into the statistical background. At low bombarding energy, the collision occurs slowly relative to the decay time of the resonance. Subsequent excitations then usually occur after the previous ones have decayed and the cross sections for coherent multiple excitation are small compared to the total ones. As the energy increases, the collision time decreases and the time available for decay of a phonon before the excitation of another also decreases. The relative importance of the incoherent contributions to the n -phonon excitation cross section thus decreases as does the relative enhancement of the total cross section over the coherent one.

There has been a good deal of discussion in recent years related to the fact that the observed width of the double giant dipole resonance deviates from the harmonic value of twice the single giant resonance width. As we have commented previously, [1] this is a natural result of the incoherent contributions to the cross section, even when the resonances themselves are purely harmonic. In fact, due to the energy dependence of the incoherent contributions to the excitation cross sections, we expect the effective n -phonon width to be energy dependent. We can estimate the effective widths here by averaging the width over the n -phonon cross sections, taking care to discount that part of each cross section that results from the decay of the previous state in the chain. That is, we take

$$\begin{aligned}\Gamma_{eff,n} &= \frac{\Gamma_{n0}\sigma_{n0} + \Gamma_{n-1,1}(\sigma_{n-1,1} - \sigma_{n0}) + \dots + \Gamma_{1,n-1}(\sigma_{1,n-1} - \sigma_{2,n-2})}{\sigma_{n0} + (\sigma_{n-1,1} - \sigma_{n0}) + \dots + (\sigma_{1,n-1} - \sigma_{2,n-2})} \\ &= \Gamma_d \frac{\sigma_{n0} + \sigma_{n-1,1} + \dots + \sigma_{1,n-1}}{\sigma_{1,n-1}},\end{aligned}\tag{18}$$

where we have used the harmonic limit assumed in the calculations to obtain the second expression. The results of this calculation for the system $^{208}\text{Pb} + ^{208}\text{Pb}$, as a function of the projectile energy, are shown in Fig. 3. We observe that the effective width of the n -phonon excitation cross section is much smaller than the harmonic value of $n\Gamma_d$ at low energy but approaches the harmonic value at high energy. Such an increase in the effective width has been observed experimentally in the case of the double giant dipole resonance. [6]

Finally, in Fig. 4, we show the differential excitation cross section that we obtain for the system $^{208}\text{Pb} + ^{208}\text{Pb}$ at 640 MeV/nucleon as a function of the excitation energy. This was obtained by summing Breit-Wigner expressions with the appropriate excitation energy and width for each of the n -phonon cross sections, again taking care to discount that part of each cross section that results from the decay of the previous state in the chain. We show only the contributions of the first three giant dipole resonances, as the higher order ones are almost invisible even on our theoretical curve. Only the first and second giant dipole resonances have been observed experimentally.

In summary, the semiclassical calculations presented here allow us to conclude that the collective-statistical theory of multiple giant resonance excitation and decay provides a theoretical basis for the energy-dependent enhancement of multiple excitation cross sections and energy-dependent effective widths observed experimentally.

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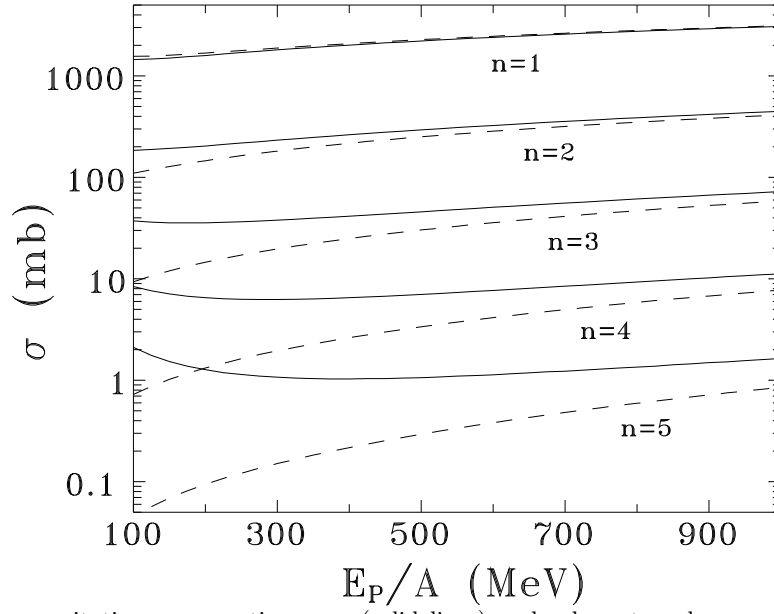


FIG. 1. Total n -phonon excitation cross sections σ_{0n} (solid lines) and coherent n -phonon excitation cross sections σ_{n0} (dashed lines) for the system $^{208}\text{Pb} + ^{208}\text{Pb}$ as a function of the projectile energy.

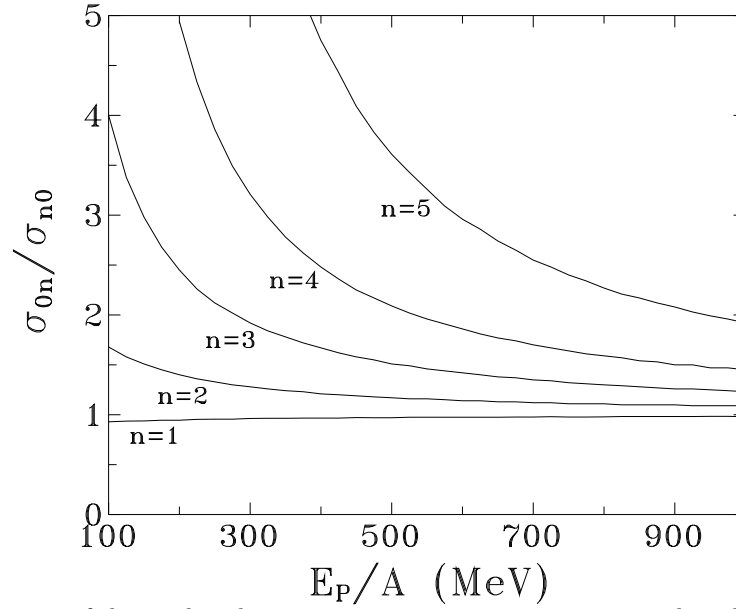


FIG. 2. Relative enhancement of the total n -phonon excitation cross section σ_{0n} over the coherent excitation cross section σ_{n0} for the system $^{208}\text{Pb} + ^{208}\text{Pb}$ as a function of the projectile energy.

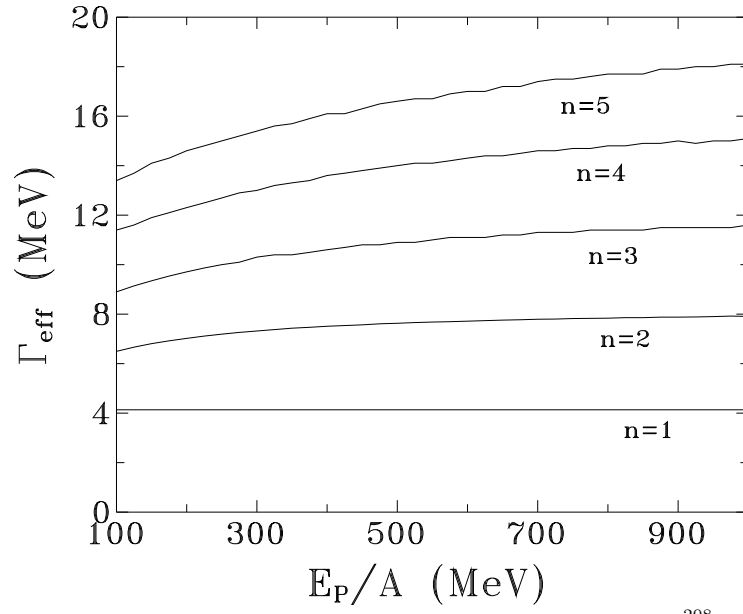


FIG. 3. Effective widths of the first five multiple giant dipole resonances of the system $^{208}\text{Pb} + ^{208}\text{Pb}$ as a function of the projectile energy.

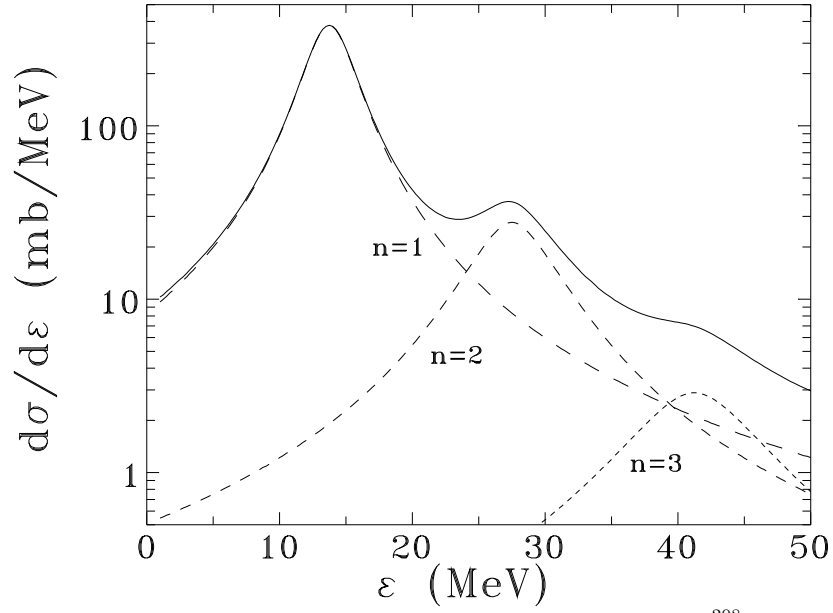


FIG. 4. Theoretical multiple giant resonance differential excitation cross section of ^{208}Pb at a projectile energy of 640 MeV/nucleon.